FAR BEYOND

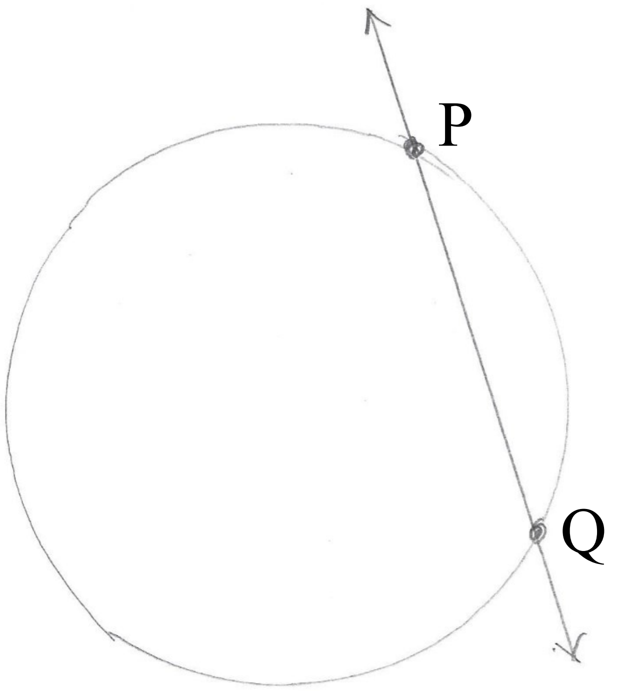
MAT122

Rate of Change

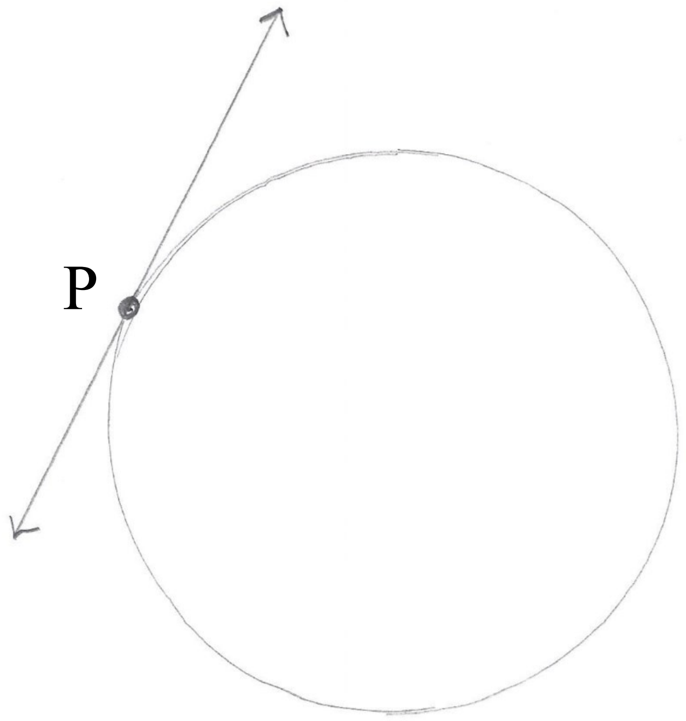
Stony Brook University

Secant vs Tangent on a Curve

Recall:



secant line

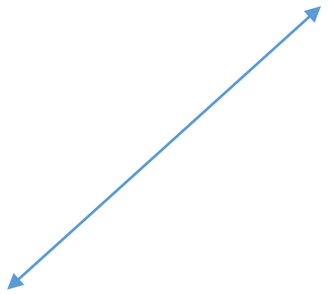
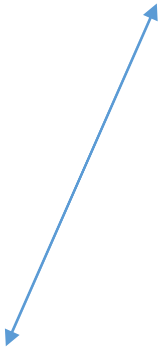


tangent line

Rate of Change of a Line - Refresher

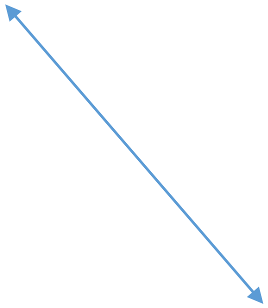
For a linear function, slope measures the **steepness**

slope is also its **rate of change**

steeper slope implies higher rate of change

(x-values and y-values are both increasing)



negative slope implies negative rate of change

(as x-values are increasing,
y-values are decreasing)

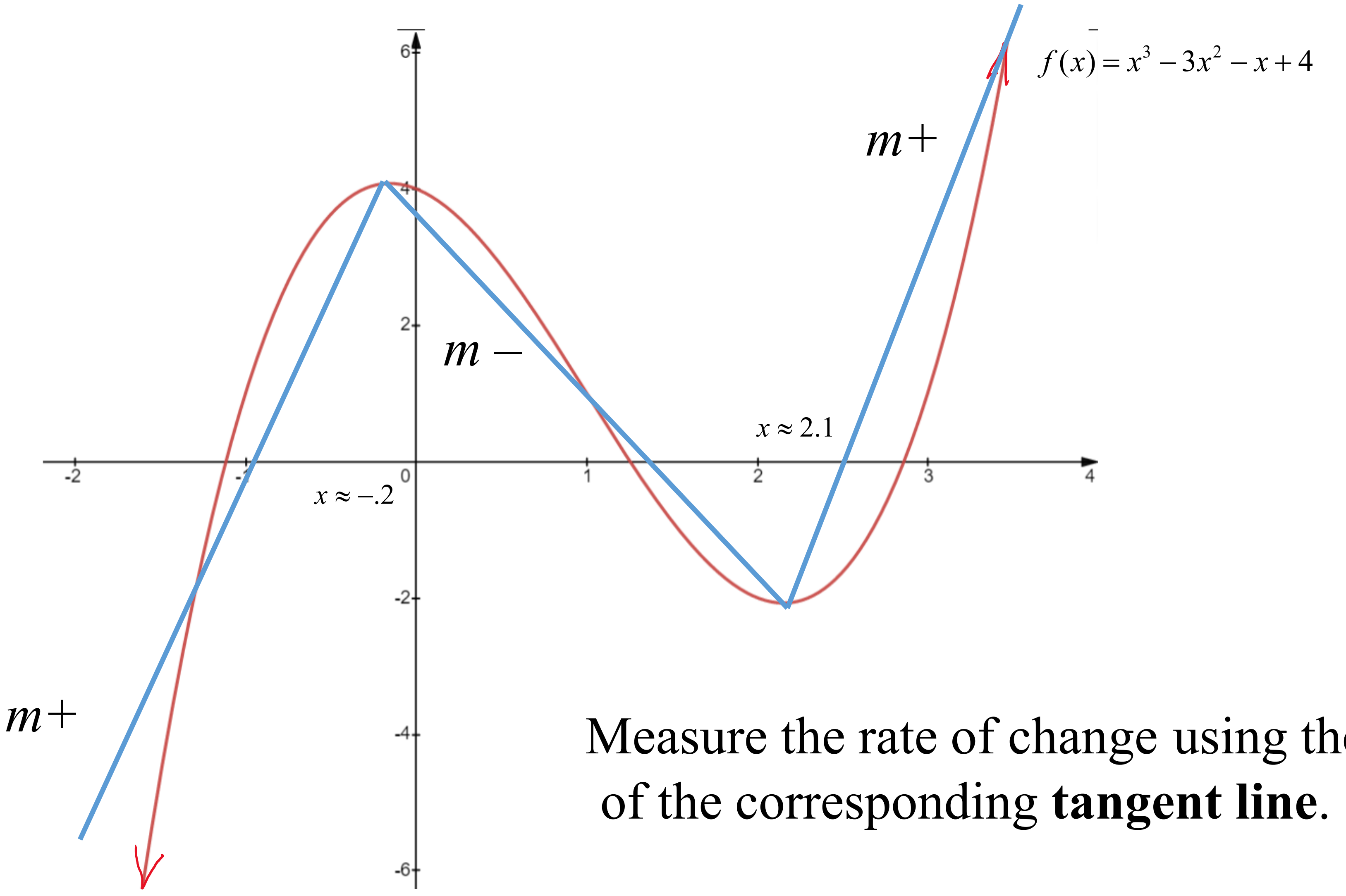
slope is 0

NO change

rate of change is **consistent** for all values of x on a line

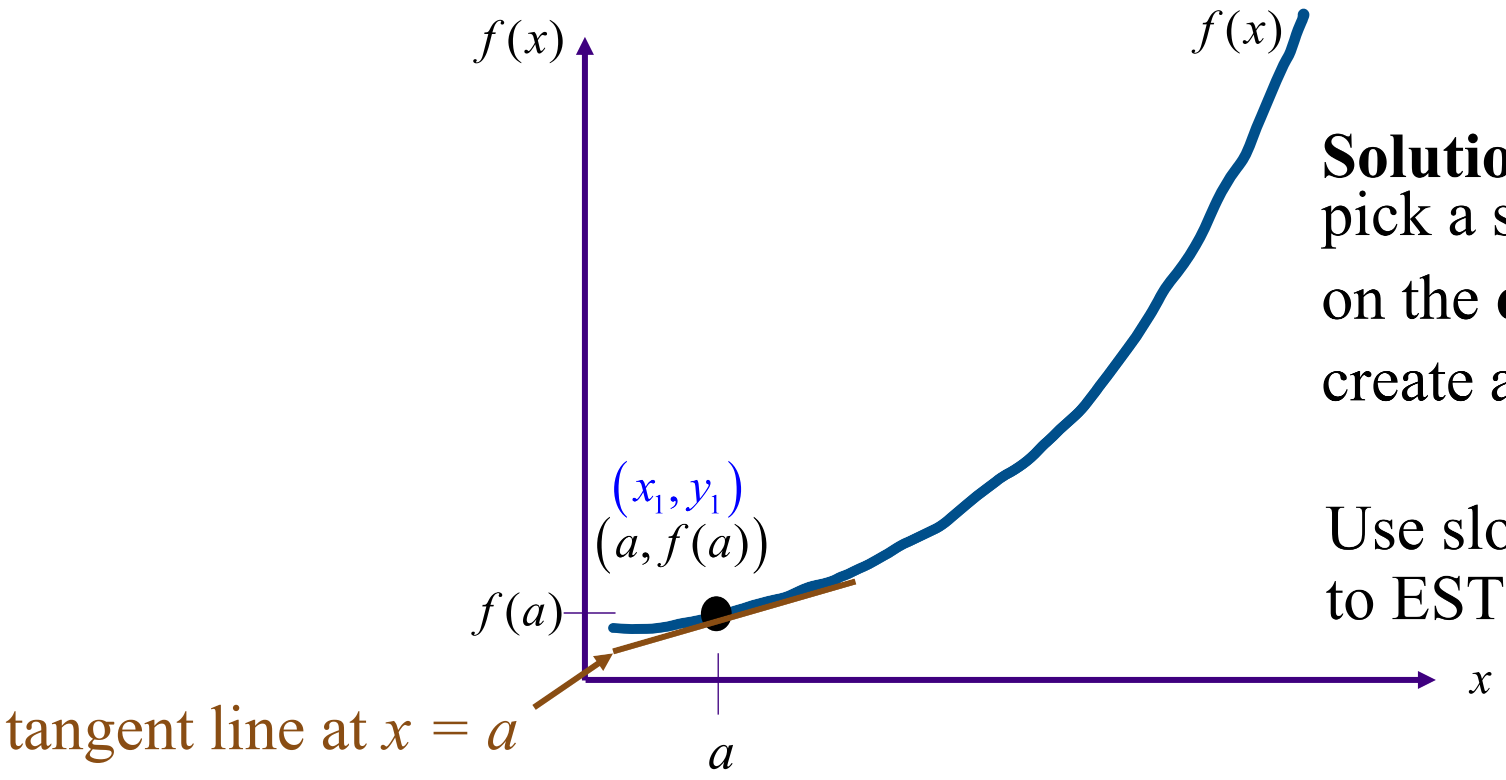
Rate of Change on a Curve

Contrary to a linear function, the rate of change can vary at different places on the curve.



Measure the rate of change using the **slope** of the corresponding **tangent line**.

Problem Finding Slope of Tangent Line



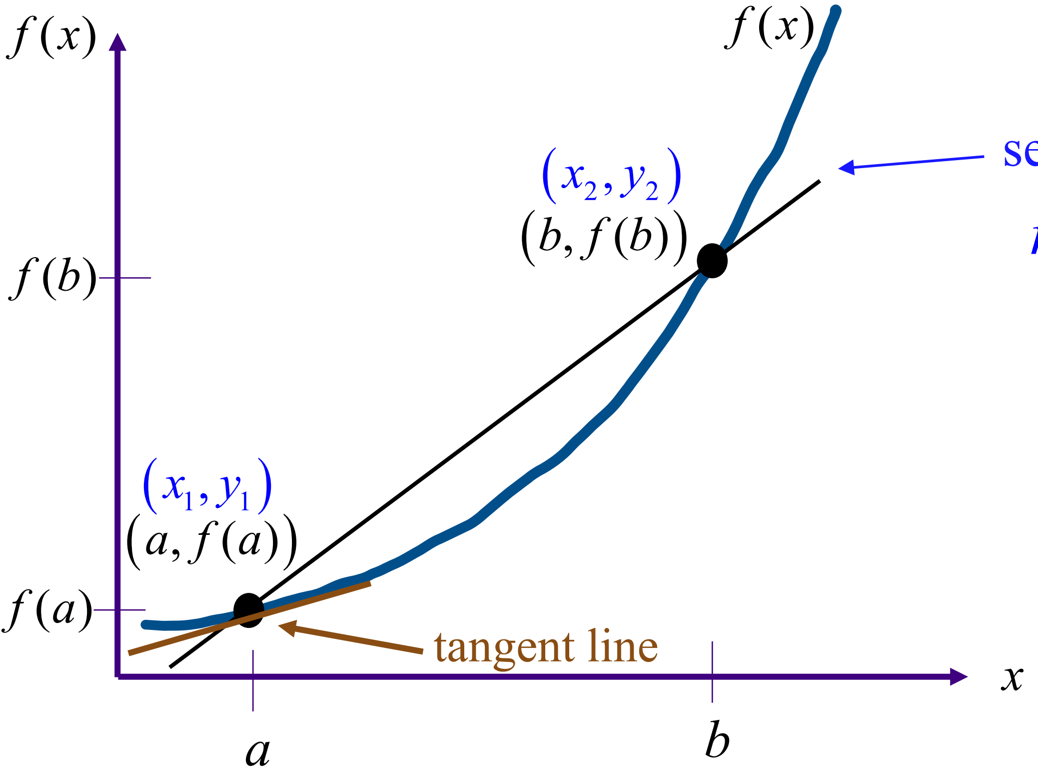
Solution:
pick a second NEARBY point
on the **curve** in order to
create a **secant** line

Use slope of **secant** line
to ESTIMATE slope of the **tangent** line

Problem: no way to find a slope if only *one* ordered pair is known

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Average vs. Instantaneous Rate of Change



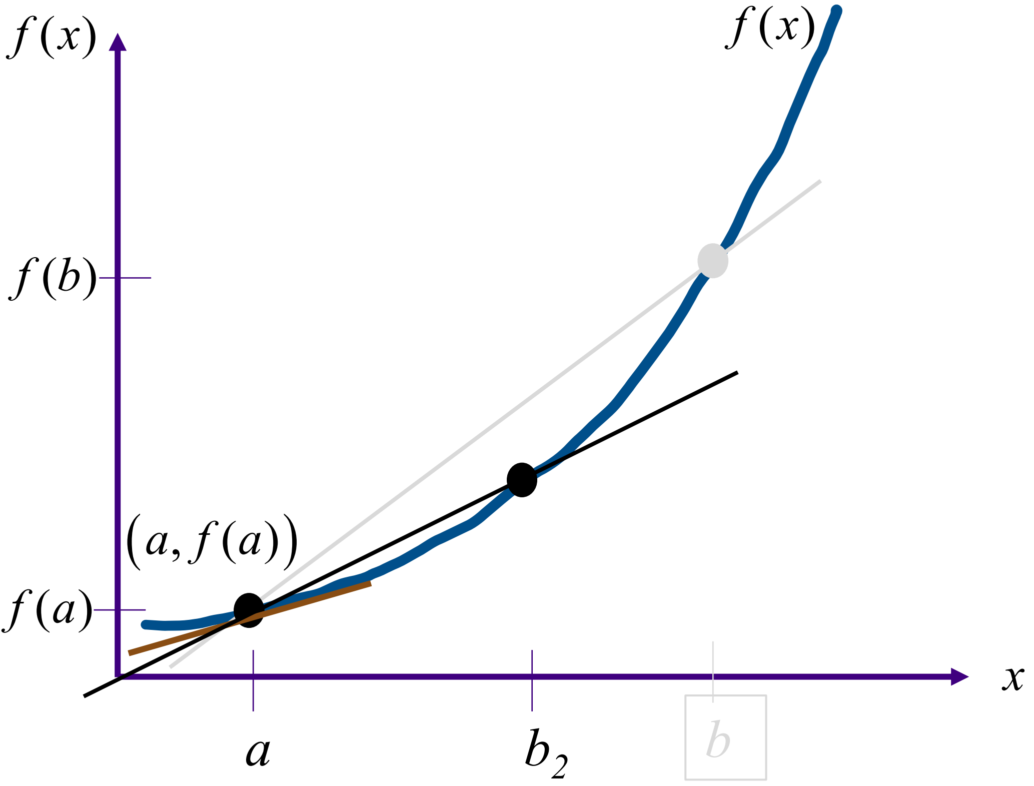
$$m_{\text{sec}} = \frac{f(b) - f(a)}{b - a}$$

slope of the secant line <u>estimates</u>
the rate of change at $(a, f(a))$

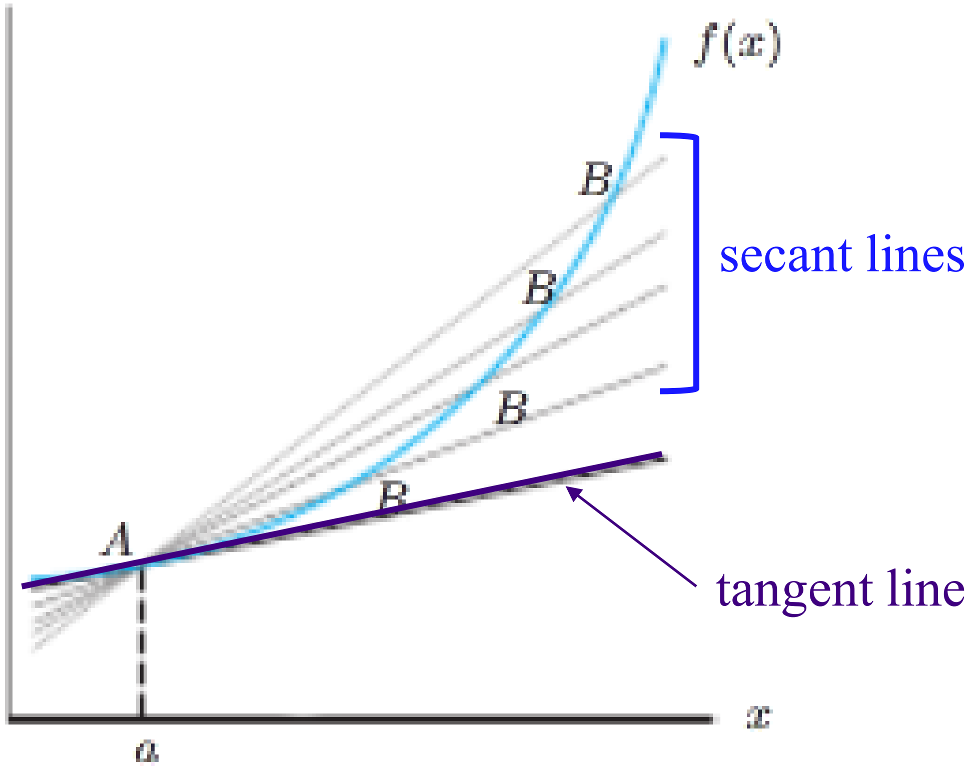
called **<u>average</u>** rate of change at that point

Fact: estimate will be better the closer that b is to a

How close is "nearby"?



slope of the new secant line
better estimates slope of tangent line



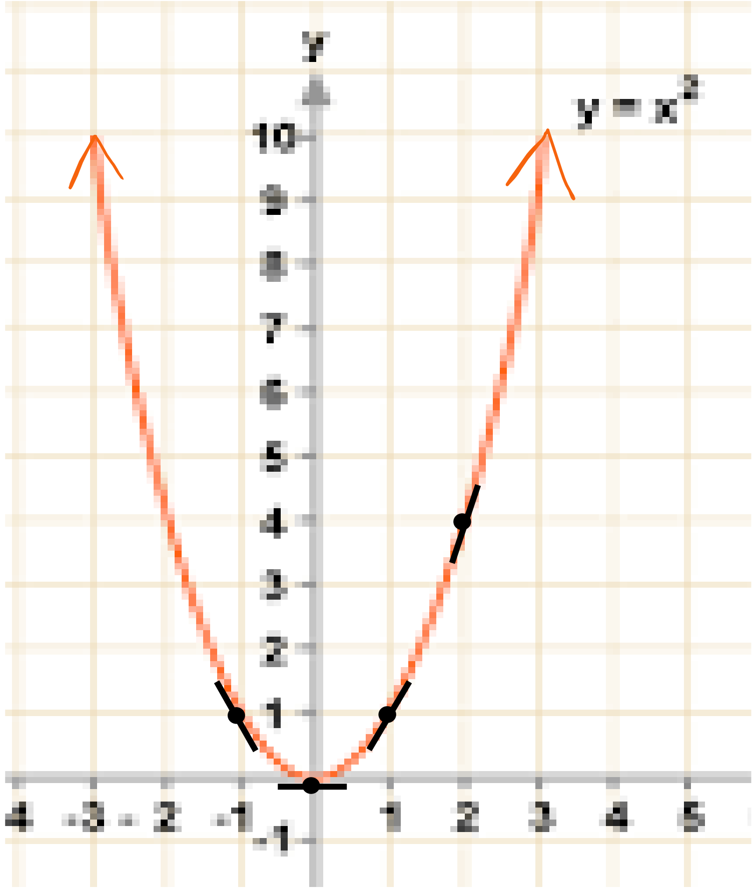
slope of tangent line is exact rate of change at $x = a$
called instantaneous rate of change at $x = a$

"f prime of a"

The **derivative** of f at $x = a$, written as $f'(a)$, is the instantaneous rate of change of f at $x = a$.

Identify Rates of Change on a Graph

The **derivative** of f at $x = a$, written as $f'(a)$, is the instantaneous rate of change of f at $x = a$.



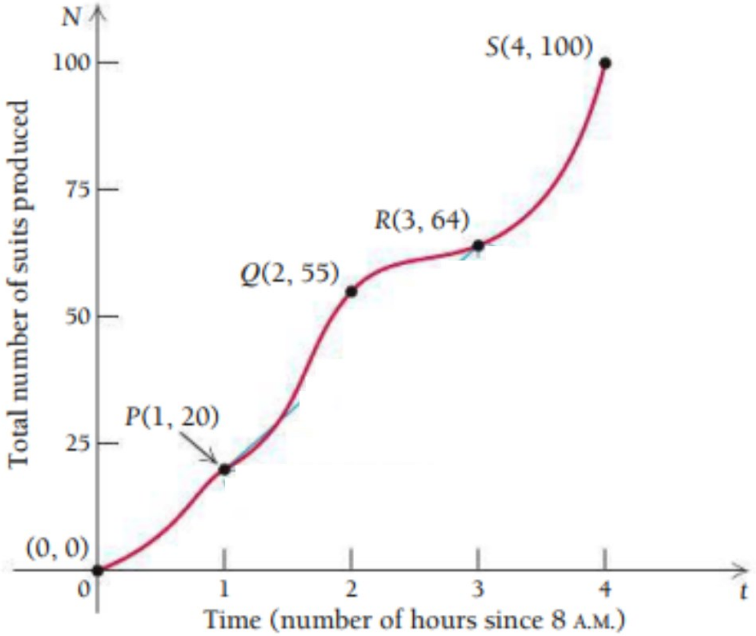
ex. Determine if the following are positive, negative or zero:

- $f'(1)$ positive
- $f'(-1)$ negative
- $f'(2)$ positive and $f'(2) > f'(1)$
- $f'(0)$ zero

Rates of Change - Application

ex. How many suits were produced between 9am and 10am?

$$55 - 20 = \boxed{35 \text{ suits}}$$



$$9:00am : t = 1$$
$$10:00am : t = 2$$

ex. What is the slope of the secant line between $t = 1$ and $t = 2$?

$$m = \frac{\Delta y}{\Delta x} = \frac{55 - 20}{2 - 1} = \frac{35}{1} = 35$$

i.e., 35 suits PER HOUR are produced

secant line shows AVERAGE RATE OF CHANGE

ex. What was the average number of suits produced per hour from 9am to 11am?

$$m = \frac{64 - 20}{3 - 1} = \frac{44}{2} = \frac{44\ suits}{2\ \ hr} \boxed{= 22 \frac{suits}{hr}}$$

More Rates of Change on a Graph

derivative $= f'(a) =$ slope of tangent line at $x = a$

ex. Given a graph, illustrate the following graphically and determine if positive or negative.

- $f'(1)$

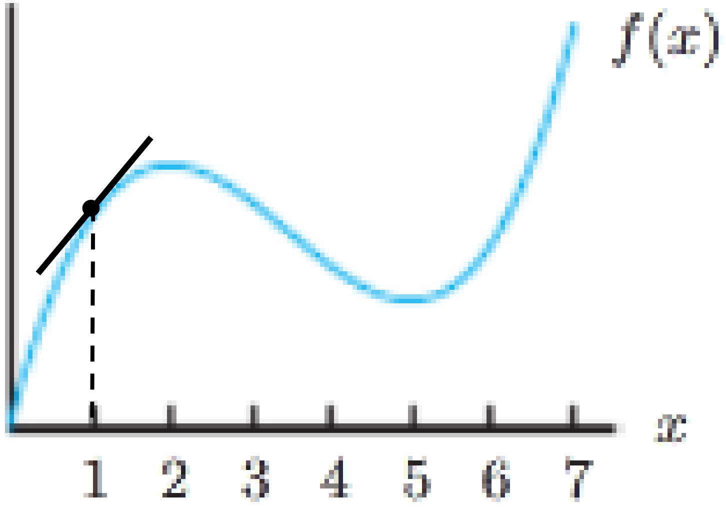
slope of tangent line @ $x = 1$

- $\dfrac{f(2) - f(1)}{2 - 1}$

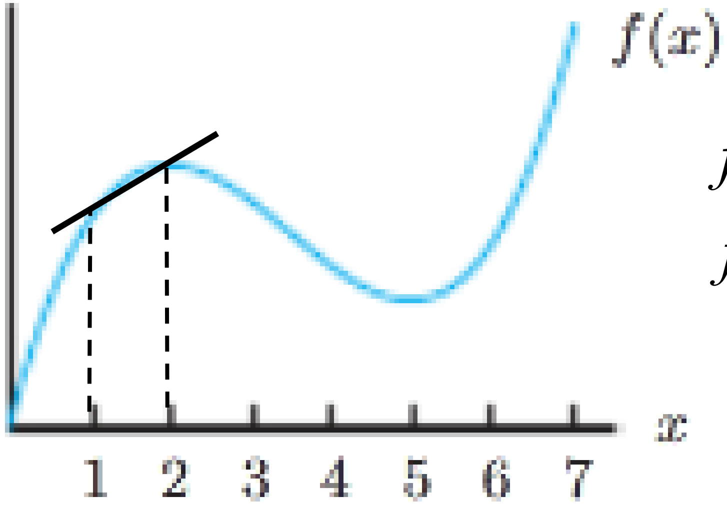
slope between $x = 1$ and $x = 2$

- $f(4) - f(2)$

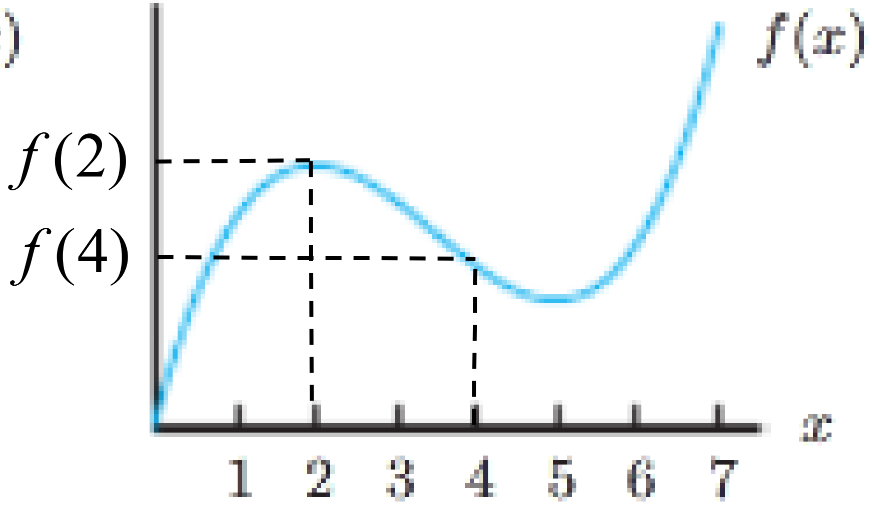
Δy



positive



positive



$f(2) > f(4)$

$\therefore f(4) - f(2)$ is negative